DO
Decision and Optimization
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Scientific Domains

Composition: 3 teams, 41 CNRS/AP/P (25 HDRs), 3 AM, 3 PostDoc, 40 Ph. D.

- **DISCO**: Diagnosis, Supervision and Control
- **ROC**: Operations Research Combinatorial Optimization and Constraints
- **MAC**: Methods and Algorithms for Control

Academic Fields:

- ✔ Automatic Control
- ✔ Applied Mathematics
- ✔ Operational Research
- ✔ Artificial Intelligence

Applied fields:

- ✔ Aerospace
- ✔ Energy
- ✔ Transport Systems
- ✔ Networks
- ✔ Biotechnologies
- ✔ Healthcare
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Scientific Objectives

✔ Diagnosis:
- Design and analyze methods for state estimation, fault detection, diagnosis and prognosis
- Continuous, discrete event and hybrid dynamical systems

✔ Optimization:
- Design and analyze methods and algorithms
- Combinatorial optimisation and constraint propagation for tasks planning and scheduling, resource allocation, polynomial optimization and moment approach

✔ Control Theory:
- Design and analyze control and supervision systems
- Robust S/C, AWU systems, hybrid S/C, observers
Life, National and International Collaborations

✔ Organization and life
- 3 autonomous teams (budget and science)
- Scientific council (DOsc) = 3 × 2 delegates + 3 team leaders: coordination, setting priorities, decision making (monthly based)
- Every-two-years internal scientific seminar on specific cross-cutting topics (robustness, optimization)

✔ National and International Collaborations
Decision and Optimization: Robustness
Robustness

✔ Robust control/diagnosis/optimization in uncertain environments
  - Robust optimization/Stochastic optimization
  - Set membership estimation for diagnosis
  - Worst-case analysis and synthesis for S/C stability and performance

□ Objective. Guaranteed certification (worst-case) of solutions for all instances in the feasible uncertainty set $\mathcal{U}$

\[
\min_x \sup_{\Theta \in \mathcal{U}} f_0(x, \Theta) \\
\text{s.t.} \quad \sup_{\Theta \in \mathcal{U}} f(x, \Theta) \leq 0, \ \forall \Theta \in \mathcal{U}
\]

\[
[x_{t_j+1}] = [x_{t_j}] + \sum_{i=1}^{k-1} h^i f^{[i]}([x_{t_j}]) + h^k f^{[k]}([\tilde{x}_{t_j}])
\]

\[
\max_{\Theta \in \mathcal{G}_\mathcal{U}} \|\Sigma(\Theta, K)\|_* \leq \max_{\Theta \in \mathcal{U}} \|\Sigma(\Theta, K)\|_* \leq \gamma_g
\]
Robustness: Identifiability, Stability Analysis, Scheduling

- Set-Membership identifiability / $\mu$-SM identifiability

$\text{SM-I} / \mu$-SM $\xrightarrow{\text{diff. alg.}}$ restricted partial injectivity / p.i. of $\phi \in \mathbb{R}^n(p_1, \cdots, p_P)$

- Mincing
- Evaluating
- Regularizing

- Robust performance analysis

$\max_{\delta \in \Delta} \| \Sigma(\delta, K) \|_* \leq \max_{\delta \in \mathcal{U}} \| \Sigma(\delta, K) \|_* \leq \gamma_g$

$\gamma_g = \min_{P(\delta) \in \mathcal{P}} \max_{\delta \in \Delta} \| \Sigma(\delta, K) \|_*$

$P(\delta) = \sum_{i=1}^N \delta_i P[i]$  

$\begin{bmatrix} 0 & P[i] \\ P[i] & 0 \end{bmatrix} + S \begin{bmatrix} A[i]' & F \\ -1 & G \end{bmatrix} < 0$

- Robust optimization for resource-constrained project scheduling with uncertain activity durations

$\min_{S_i^h, x_{ij}, \rho} \rho$

$\rho \geq S_{n+1}^h - S_{n+1}^*(p_h) \quad h = 1, \cdots, |\mathcal{P}|$

$S_j^h \geq S_i^h + p_i^h - M(1 - x_{ij}), (i, j) \in V^2$

$S_i^h \geq 0 \quad \forall i \in V, \rho \geq 0, \ X \in \mathcal{X}$
Decision and Optimization: Optimization
✅ Polynomial Optimization and Generalized Moment Problem (GMP)

\[
\begin{align*}
\min_{\mu \in \mathcal{M}(K)} & \quad \int f_0 d\mu \\
\text{s.t.} & \quad \int f_j d\mu \geq b_j 
\end{align*}
\]

- Measure Theory
- Duality and SDP relaxations
- Putinar’s positivity certificates and SOS representations
- Applications to impulsive optimal control, inner approximations of robust stability and attraction regions

✅ Combinatorial Optimization Problems (COP)
- Multi-objective, uncertainties, human factors
- MILP formulations, CSP, graphs
- Scheduling, transportation, resource allocation
Polynomial and Multi-Objective Optimization

\[ f^* = \min_{x \in K} f(x) \quad K = \{ x \in \mathbb{R}^n : g_j(x) \geq 0, j = 1, \ldots, m \} \subset \mathbb{R}^n \text{ compact} \]

\[ f, g_j \in \mathbb{R}[x] \]

Lagrangian Relaxation

- SDP relaxations \((\gamma_d)_{d \in \mathbb{N}}, n \text{ small}\)

\[ \gamma_d \xrightarrow{f.c. \ d \uparrow} f^* \quad \text{et} \quad -g_j \text{ Conv and} \quad \nabla^2 f(x^*) > 0 \Rightarrow \gamma_1 = f^* \quad \gamma_d = f^* \]

- LP relaxations \((\theta_d)_{d \in \mathbb{N}}, n \text{ high}\)

\[ \theta_d \xrightarrow{d \uparrow \infty} f^*, \text{ conditioning} \downarrow \]

- Hybrid relaxations \((\rho_d)_{d \in \mathbb{N}}\)

\[ \rho_d \xrightarrow{f.c. \ d \uparrow} f^*, f, -g_j \text{ SOS} - \text{Conv} + \text{Slater} \Rightarrow \rho_1 = f^*, K \subseteq \{ 0, 1 \} \Rightarrow \text{finite convergence} \]

\[ f(x, y) = \min \{ f_1(x, y), f_2(x, y) \} \]

\[ f_1 = -5.09 \quad 0 \quad (1.94, 4.92) \]
\[ f_1 = -3.24 \quad 1 \quad (1.83, 4.5) \]
\[ f_1 = -1.28 \quad 2 \quad (0.2) \]

\[ f_1 = -3.92 \quad 0 \quad (2, 3) \]
\[ f_1 = -2.92 \quad 1 \quad (1.3) \]
\[ f_1 = -1.28 \quad 2 \quad (0.2) \]

\[ f_1 = -4.56 \quad 0 \quad (2.4) \]

\[ f_2 \]

\[ x_1 \geq 2 \]
\[ x_1 \leq 1 \]
\[ x_2 \leq 3 \]
\[ x_2 \geq 4 \]

\[ \min_{x_1, x_2} -1.00x_1 - 0.64x_2 \]
\[ \min_{x_3} \]
\[ x_3 \]
\[ 50x_1 + 31x_2 \leq 250 \]
\[ 3x_1 - 2x_2 \geq -4 \]
\[ x_1 + x_3 \leq 2 \]
\[ x_1, x_2 \geq 0 \text{ integer} \]
\[ x_3 \in \{ 0, 1, 2 \} \]

\[ \text{Visite ENS 2015 Decision and Optimization} \]
Hybrid Systems
Decision and Optimization: Hybrid Systems

✔ Continuous time (flow) and discrete time (jumps)
  (differential, difference and logic equations)

✔ hybrid S/C for linear and nonlinear systems

✔ Hybrid formalism and systems with impacts

✔ Hybrid formalism and limited information
  quantified I/O, sampling

✔ Mode automata: Continuous and discrete dynamics

✔ Abstraction of the continuous dynamics for diagnosis

✔ Aging laws for prognosis

✔ Uncertain continuous dynamics

\[ \dot{x}(t, p_2) = f_2(x(t, p_2), u(t), p_2) \]
\[ y(t, p_2) = h_2(x(t, p_2), p_2) \]
\[ x(t_0, p_2) = x_0 \in X_0 \]
\[ p_2 \in P_2 \subset U_{p2}, \ t_0 \leq t \leq T \]
Global Exponential Stability via Hybrid S/C

\[
\begin{align*}
\dot{x}_c &= A_c x_c + B_c y \\
\dot{\tau} &= 1 - dz \left( \frac{\tau}{\rho} \right) \\
u &= C_c x_c + D_c y
\end{align*}
\]

Reset rules:
\[
x^+ = G x
\]

\[
\begin{align*}
\dot{x}_p &= A_e x_p + B_e u + Ly \\
\dot{y}_p &= C_e x_p + D_e u
\end{align*}
\]

\[
\begin{align*}
\check{\dot{x}} &= A_e \check{x} + B_e u + Ly \\
\check{\dot{y}} &= C_e \check{x} + D_e u
\end{align*}
\]

\[
x = \begin{bmatrix} \check{x}^T & x_c^T & e^T \end{bmatrix}^T
\]

\[
\begin{align*}
\dot{x}(t) &= A x(t) \\
\dot{\tau}(t) &= 1 - dz \left( \frac{\tau}{\rho} \right) \quad \text{if } x \in F \text{ or } \tau \in [0, \rho] \\
x^+ &= G x \quad \text{if } x \in J \text{ and } \tau \in [\rho, 2\rho]
\end{align*}
\]

\[
A = \{0\} \times [0, 2\rho], \quad \rho \in (0, \rho^*) \text{ is GES}
\]

\[
F \text{ and } J \text{ defined by Lyapunov/}(\check{x}_p, x_c)
\]

- I/O behaviour for different \(G\)
- \(x_c(0, 0) = 0, \quad x_p(0, 0) = -\begin{bmatrix} 1 & 1 \end{bmatrix}^T\)
- State-feedback/observer-based output feedback
- Effects of the dynamics of the observer: Over-shoot

Visite ENS 2015 Decision and Optimization
Active and Passive Diagnosis of Hybrid Models

✓ Automata with enriched behavior

- ARR Analytic Redundancy Relations

\[ \rho_{ci}^p(n) = \Omega_i^p Y_i^p(n) - \Omega_i^p L_i^p(A_i, B_i, C_i, D_i) U_i^p(n) \]

- Residuals generation

\[ r_{ij} = \text{Boolean consistency indicator} \]

- Mode signature

\[ \text{Sig}(q_j) = \begin{bmatrix} S_{j/1}^T, \ldots, S_{j/j}^T, \ldots, S_{j/m}^T \end{bmatrix}^T \]

✓ Diagnoser

- State machine based on observations
- Estimation of actual mode
- Test of diagnosability
- On-line diagnosis
- Identify occurred faults
- Active diagnosis
Constraint Satisfaction Problems

Demo: M.-J. Huguet, "Efficient algorithms for multimodal path computation"
Propagation and Analysis of Conflicts for Sequencing Constraints

SAT solvers and constraints propagation

✔ AtMostSeqCard constraint
Sequencing constraint:
Not more than 2 air conditioning for a sub-sequence of 3 vehicles (vehicles assembly)

✔ Propagation of sequencing constraints
Linear optimal arc consistency algorithm (O(n)) AtMostSeqCard(2/5/4)

✔ Accelerating methods for the tree search via conflicts learning
Explain non satisfiability of constraints and conflict-driven clause-learning \( \neg x_2 \land \neg x_3 \land \neg x_4 \)
Meta-Diagnosis Reasoning

✔ Model-based diagnosis
- Knowledge about the system \( SD \) (predicate logic) + Components \( CPS \) + Observations \( OBS \) + Algorithm \( ALG \)
- Diagnosis: Health status \( (Ab(c_i) \text{ or } \neg Ab(c_i)) \) of components satisfying constraints \( DS \land OBS \) with \( ALG \)

✔ Introduction of the meta-diagnosis problem
- How the solution of \( (SD, OBS, CPS) \) with \( ALG \) could be faulty?
- Analyze required properties: correction - completion - covering

✔ Design and solution of a meta-diagnosis problem
- Choose assumptions, meta-components \( MeCPS \)
- MEDITO: \( MeDP = (MeSD, MeCPS, MeOBS) \)
- Solution of \( MeDP \) by successive CSPs oriented by minimal conflicts
Embedded Control, Diagnosis and Optimization

- Limited capacity (CPU, memory, format)
- Real-time and communication constraints
- Rigorous computing (symbolic-numeric objects)
- Algorithms with guaranteed convergence/performance
- Stochastic modelling

Formation/fleet of vehicles

- Limited or non standard information structure for control and estimation systems
- Non standard information structure
- Distributed actuators and sensors
- Distributed functional organization
- Distributed Optimization

LP/LMI/SDP/NLP = embedded technology?
Impulsive Space RDV problem

Some results and theoretical stakes

- Open-loop or closed-loop/direct or indirect Architecture for guidance/control

- Direct methods
  Robustness, optimality / uncertainties and GNC systems errors
  Collision avoidance and stable orbits

- Indirect methods
  Impulsive OC (Primer Vector), PMP
  Orbital perturbations

Technological and numerical stakes

- Toolbox PolyRDV 1.0®
- Embedded algorithm (LEON - 32-bit - SPARC-V8 RISC)
- Certified optimal trajectories via rigorous computations
- Collaborations: CNES (MAC+ROC), ADS, Region, TAS

Visite ENS 2015 Decision and Optimization
Some Facts and Figures: Industrial Partnerships

✔ Space Systems
- ~10 PhDs (CNES/*, CNRS/*, CIFRE)
- ~10 direct scientific collaborations

✔ Aeronautics
- AIRSYS, CORAC, OCKF, SIRASAS
- 1 CIFRE, 1 direct PhD grant

✔ Transportation Systems
- ANR, OSEO
- 3 CIFRE (1 SNCF - 2 MOBIGIS)
- Demo: M.-J. Huguet, "Efficient algorithms for multi-modal path computation"

✔ Healthcare
- ONCOMATE Project (IMRCP, LBB, INNOPSIS SA, MNBT)
- 1 ANR BIOSTEC INNODIAG (DENDRIS, ITAV, ICR, MNBT)
Internships for 2015

✔ Diagnosis and hybrid models:
  - Health Monitoring of a system based on adaptive models
  - Learning of temporal patterns for complex systems: Application to mobile robots

✔ Control, space rendezvous, computer science:
  - Convergence proof of an algorithm for space rendezvous planning
  - Robust analysis of an impulsive control algorithm for space rendezvous

✔ Scheduling and combinatorial optimization:
  - Scheduling with multiple energy sources
  - Design and development of a constraint model for optimization of tests of planning
  - Solution of a crossdocking problem
  - Scheduling of an aircraft assembly chain
  - Algorithms for alternate path computation in multimodal graphs
  - S-coloration of graphs