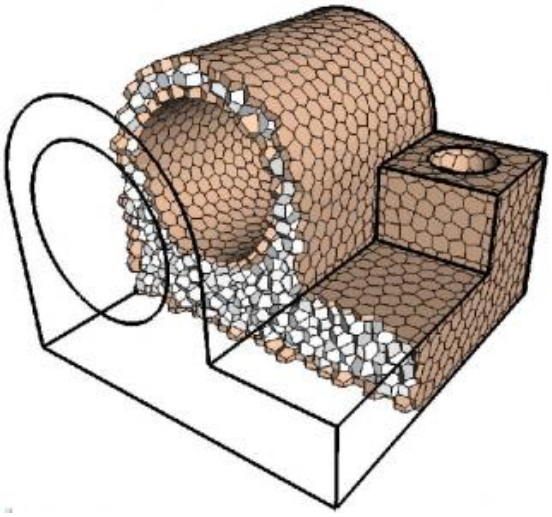
Iterative H-to-V conversion
for mesh generation



- **Theme:** Computational Geometry
- **Lab:** INRIA Nancy Grand-Est, Nancy, France
- **Teams:**
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General presentations

This project takes place in the frame of the ERC starting grant GOODSHAPE (http://alice.loria.fr/goodshape), on optimal sampling and mesh generation. Given a 3D shape, the optimal sampling problem consists in positioning a set of points inside the shape in a way that minimizes the so-called quantization noise power [DFG99]. In the frame of project GOODSHAPE, we proved the continuity of the objective function and proposed several generalizations [LY10]. Minimizing the quantization noise power requires to accurately construct some geometric structures (restricted Voronoi diagram and clipped Voronoi diagram, depicted in the figure). The computation of these structures depends on some elementary functions, called geometric predicates, that analyse the relative locations of some points. In order to ensure that the combinatorics are consistent, these functions need to be resistant to degeneracies, such as a point that is exactly located on a plane. The goal of this project is to design an algorithm to quiclky compute the Voronoi cells (the polyhedra depicted on the figure).

Goals of the training period

We consider a polytope Ω defined by the intersection of n half-spaces Π_i (H-form) and we want to enumerate the vertices, edges and faces of Ω (V-form). Each half-space Π_i is defined by the bisector of two points p_0, p_i by $\Pi_i = \{p/d(p,p_0) < d(p,p_i)$. The goal of the project is to define the data structures and algorithms to find all the vertices and faces of $\cap_i \Pi_i$.

- Given $\Omega_k = \cap_{i=1}^{k} \Pi_i$ and an additional half-space Π_{k+1}, characterize the existence of an intersection between an edge of Ω_k and Π_{k+1}. In particular, the configuration can be characterized in terms of whether p_{k+1} is located in the circumscribed sphere of a terahedron [EM90] ;
- Design a combinatorial data structure for the algorithm. For instance, the boundary of Ω_k can be represented in dual form, by a triangulation.
- (Optional) implement the algorithm in C++.

Required skills

Taste for mathematics and algorithms. Enthusiasm is a must!

Possible continuation

There are several possibilities for continuing this work (Master, Ph.D.), in the frame of project GOODSHAPE.

References

[EM90] Herbert Edelsbrunner and Ernst Peter Mucke. Simulation of Simplicity: A Technique to Cope with Degenerate Cases in Geometric Algorithms *ACM Transactions on Graphics*, 1990.

[DFG99] Qiang Du, Vance Faber and Max Gunzburger. Centroidal Voronoi Tesselations: Applications and Algorithms *SIAM review*, 1999

[LY10] Bruno Lévy and Yang Liu. L_p Centroidal Voronoi Tesselation and its Applications *ACM Transactions on Graphics - SIGGRAPH conf. proc.*, 2010.