The Kadison-Singer Problem

Marcus Spielman and Srivastava managed to prove this family to be interlacing, and that it still holds for any kind of forest.

Our work consisted in proving a similar result for non-bipartite Ramanujan graphs. We found that the problem could be reduced to finding the maximum root of \( t \mapsto \text{MAP}_G \prod_{x \in V} (t - \sum_{y \in x} (y_v - 1))^2 \) for all \( V, E \) of the univariate polynomial \( t \mapsto b(t e - x) \) has only real zeros.

Hence, we managed to prove that the family of partial Eulerian polynomials are interlacing families, which is a method used to bound the maximum root of a family of polynomials, in conjunction with theorems and results on hyperbolic polynomials. We present some examples of applications of this method.

Hyperbolic polynomials

A hyperbolic polynomial is a polynomial whose non-zero terms all have the same degree. A homogeneous polynomial \( h(x) \in \mathbb{R}[x_1, \ldots, x_n] \) is hyperbolic with respect to a vector \( e \in \mathbb{R}^n \) if \( h(e) \neq 0 \), and if for all \( x \in \mathbb{R}^n \) the univariate polynomial \( t \mapsto h(te - x) \) has only real zeros.

Spanning trees

Brändén proved in [5] a sufficient condition for graphs to have disjoint spanning trees.

For each \( T \in \mathcal{T} \) the set of spanning trees of \( G \), if \( a(T) = \prod_{x \in T} x \), then the polynomial \( P = \sum_{T \in \mathcal{T}} a(T) \) is hyperbolic with respect to all the ones vector. Using the method of interlacing families to this problem, we can prove the following.

References


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